

A Mathematical Model on Jeffrey Fluid Flow through an Overlapping Stenosis

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ABSTRACT: Steady incompressible Jeffrey fluid flow through a uniform tube with an overlapping stenosis is investigated. Assuming the stenosis to be mild, the equations governing the flow of the proposed model have been solved and closed form solutions are obtained for velocity, pressure drop, volumetric flow rate, resistance to the flow and wall shear stress. The resistance to the flow increases with the height and length of the stenosis. The effects of other parameters on the flow characteristics also have been studied.

KEYWORDS: Overlapping stenosis, Jeffrey fluid parameter, wall shear stress.

I. INTRODUCTION

The term 'Stenosis' refers to narrowing of any body passage, tube or orifice, and is a most frequently occurring cardiovascular disease in arteries. Stenosis or arteriosclerosis is the abnormal growth in the arterial wall thickness that develops at various locations of the cardiovascular system under diseased conditions, which results into serious consequences (myocardial infarction, cerebral strokes, argina pitoris, etc.). Stenosis is caused by the impingement of extravascular masses or due to intravascular atherosclerotic plaques which develop at the wall of the artery and protrude into the lumen. Regardless of the cause, once an obstruction has developed, it results into significant changes in blood flow, pressure distribution, wall shear stress and the flow resistance. In the region of narrowing arterial constriction, the flow accelerates and consequently the velocity gradient near the wall region is steeper due to the increased core velocity resulting in relatively large shear stress on the wall even for a mild stenosis.

Owing to its serious concern, a major research work is being done over all parts of the world for early detection and prevention from being affected by the cardiac attack and for the diagnosis of the heart diseases.

Many researchers have studied the effect of stenosis on the flow characteristics (Forrester and Young [1], Deshpande et al. [2]) by assuming blood as a Newtonian fluid. Some experimental studies show that blood behaves like a non-Newtonian fluid at low shear rates in tubes of small radius (Huckaba and Hahn [3]). Hence many investigators carried out a good number of studies on various aspects associated with blood flow through stenosed vessels by proposing different non-Newtonian fluid model (Chaturani and Ponalagusamy [4], Sapna Ratan Shah [5]).

Akbar and Nadeem [6] discussed the Jeffrey fluid flow through a tapered stenosed artery with variable viscosity. The flow of blood through an artery with an axially asymmetric stenosis by considering two-layered macroscopic model was studied by Ponalagusamy [7].

Some stenosis may develop in series, multiple, irregular or overlapping. The pressure gradient in radial and axial direction of the artery in the presence of overlapping stenosis was analyzed by Layek et al., [8]. Srivastava et al., [9] developed a mathematical model in the presence of overlapping stenosis by treating blood as a Casson fluid.

In view of the above, a mathematical model for blood flow through a tube with overlapping stenosis is developed. For this model, blood is considered as Jeffrey fluid. The governing equations of Jeffrey fluid are solved by assuming mild stenosis. The effects of various parameters on the flow characteristics have been studied.

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II. RELATED WORK

Consider the steady incompressible Jeffrey fluid flow through a uniform tube with overlapping stenosis as shown in Fig-1.

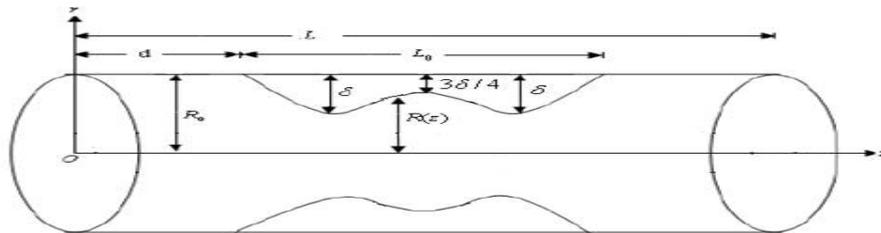


Fig- 1: Geometry of the overlapping stenosis in an artery.

The radius of the tube is taken as, (Srivastava et al. [8])

$$h = \frac{R(z)}{R_0} = 1 - \frac{3}{2} \left(\frac{\delta}{R_0 L_0^4} [11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4] \right) \quad d \leq z \leq d + L_0,$$

$$= 1, \text{ otherwise.} \quad (1)$$

Where $R(Z)$, $R_0(Z)$ is the radius of the tube with and without stenosis, L_0 is the length of the stenosis and d indicates stenosis location, the maximum height of the stenosis δ appears at two positions $Z = d + \frac{L_0}{6}$, $Z = d + 5\frac{L_0}{6}$, respectively. The critical height at $Z = d + \frac{L_0}{2}$, from the origin.

The constitutive equations for an incompressible Jeffrey fluid are

$$T = -pI + S \quad (2)$$

$$S = \frac{\mu}{1+\lambda_1} \left(\frac{\partial \gamma}{\partial t} + \lambda_2 \frac{\partial^2 \gamma}{\partial t^2} \right) \quad (3)$$

Where T and S are Cauchy stress tensor and extra stress tensor respectively, p is the pressure, I is the identity tensor, λ_1 is the ratio of the relaxation to retardation times, λ_2 is the retardation time, μ is the dynamic viscosity and $\dot{\gamma}$ is the shear rate.

The governing equations for the present problem are as follows

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

$$\rho \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) u = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rr}) + \frac{\partial}{\partial z} (S_{rz}) - \frac{S_{\theta\theta}}{r} \quad (5)$$

$$\rho \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) w = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rz}) + \frac{\partial}{\partial z} (S_{zz}) \quad (6)$$

Where the extra tensor components are

$$S_{rr} = \frac{2\mu}{1+\lambda_1} \left(1 + \lambda_2 \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \frac{\partial u}{\partial r} \right) \quad S_{rz} = \frac{\mu}{1+\lambda_1} \left(1 + \lambda_2 \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right) \right)$$

$$S_{zz} = \frac{2\mu}{1+\lambda_1} \left(1 + \lambda_2 \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \frac{\partial w}{\partial z} \right)$$

$$S_{\theta\theta} = \frac{2\mu}{1+\lambda_1} \left(1 + \lambda_2 \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \frac{u}{r} \right)$$

Further, u and w are the velocities in r and z -directions respectively.

$$\bar{z} = \frac{z}{L}, \bar{\delta} = \frac{\delta}{R_0}, \bar{R} = \frac{R}{R_0}, \bar{P} = \frac{P}{\frac{\mu UL}{R_0^2}}, \bar{u} = \frac{u}{U}, \bar{w} = \frac{L}{U\delta} w, Re = \frac{\rho R_0 U}{\mu} \quad (7)$$

Introducing the above non dimensional variables in Eq.(4)-(6),with mild stenosis assumption takes the form

$$\frac{\partial p}{\partial r} = 0 \quad (8)$$

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$$\frac{\partial p}{\partial z} = \frac{1}{1+\lambda_1} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) \quad (9)$$

Since the flow is assumed in the z-direction only, hence the velocity component in r- direction is zero. The corresponding non dimensional boundary conditions are given by

$$\frac{\partial w}{\partial r} = 0 \text{ at } r = 0 \quad (10)$$

$$w = 0 \text{ at } r = h \quad (11)$$

Solving the equation (9) using the boundary conditions Eqs. (10) and (11), yields

$$w = \left(\frac{h^2 - r^2}{4} \right) (1 + \lambda_1) \frac{dp}{dz} \quad (12)$$

The volumetric flow rate is defined by

$$Q = 2\pi \int_0^h wr \, dr \quad (13)$$

After integrating,

$$Q = \frac{\pi h^4}{8} (1 + \lambda_1) \frac{dp}{dz} \quad (14)$$

$$\frac{dp}{dz} = \frac{8Q}{\pi h^4 (1 + \lambda_1)} \quad (15)$$

When $\lambda_1 = 0$, the equations indicate the Newtonian fluid.

The pressure drop Δp in the presence of stenosis is obtained by integrating Eq. (15), as

$$\Delta p = \int_0^L \frac{dp}{dz} \, dz = \int_0^L \frac{8Q}{\pi h^4 (1 + \lambda_1)} \, dz \quad (16)$$

The flow resistance $\bar{\lambda}$, is

$$\begin{aligned} \bar{\lambda} = \frac{\Delta p}{Q} &= \frac{1}{Q} \int_0^L \left(\frac{8Q}{\pi h^4 (1 + \lambda_1)} \right) \, dz \\ &= \left\{ \int_0^d \left(\frac{8}{\pi h^4 (1 + \lambda_1)} \right) \, dz + \int_d^{d+L_0} \left(\frac{8}{\pi h^4 (1 + \lambda_1)} \right) \, dz + \int_{d+L_0}^L \left(\frac{8}{\pi h^4 (1 + \lambda_1)} \right) \, dz \right\} \end{aligned} \quad (17)$$

The expression for wall shear stress is given by

$$\tau_h = -\frac{h}{2} \frac{dp}{dz} \quad (18)$$

III. EXPERIMENTAL RESULTS

In order to explore the effects of various parameters on flow resistance ($\bar{\lambda}$) and wall shear stress (τ_h), these quantities are computed numerically and analyzed through the graphs (Figs 2-8). For better understanding of the results, the following data is considered [7] ($d = 0.2, L_0 = 0.4, Q = 0.1, \lambda_1 = 0.2$).

Figs. 2-4 show the effects of various parameters on flow resistance. From these figures it is noticed that the resistance to the flow increases with the height, length of the stenosis and length of the tube but the resistance to the flow decreases with Jeffrey fluid parameter.

The variation of pressure drop for different values of λ_1, δ is shown in Figs. 5-6. It is observed that the pressure drop increases with the height of stenosis but it decreases with Jeffrey fluid parameter.

Figs.7-8 illustrates the influence of height of stenosis and Jeffrey fluid parameter on the wall shear stress. The wall shear stress increases with the height of stenosis but decreases with Jeffrey fluid parameter.

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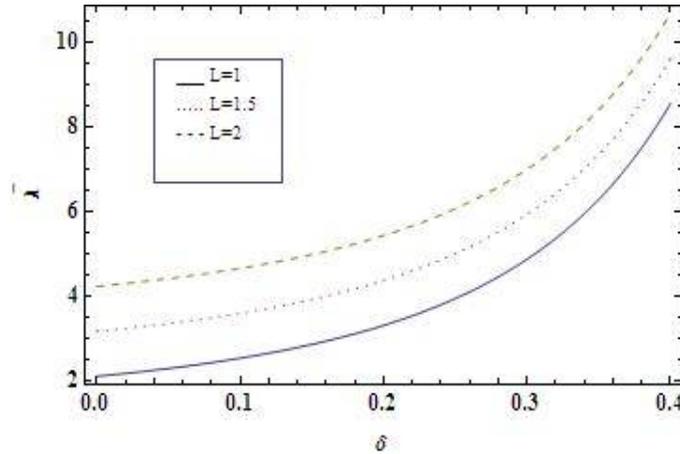


Fig 2: Variation of flow resistance $\bar{\lambda}$ with δ for different L ($d = 0.2, L_0 = 0.4, Q = 0.1, \lambda_1 = 0.2$)

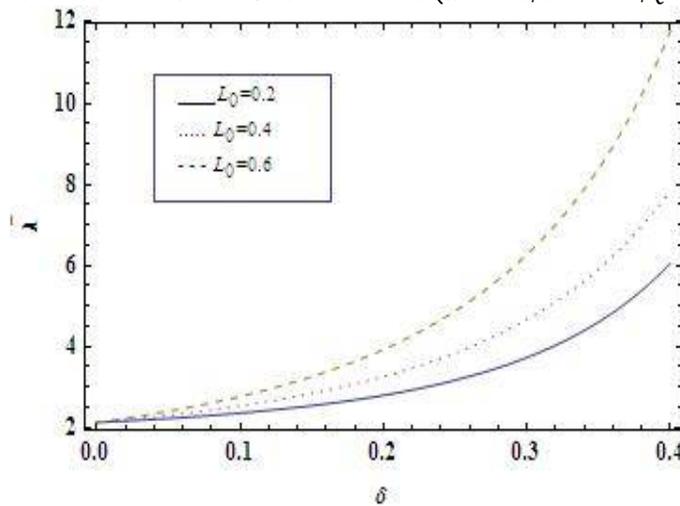


Fig 3: Variation of flow resistance $\bar{\lambda}$ with δ for different L_0 ($d = 0.2, L = 1, Q = 0.1, \lambda_1 = 0.2$)

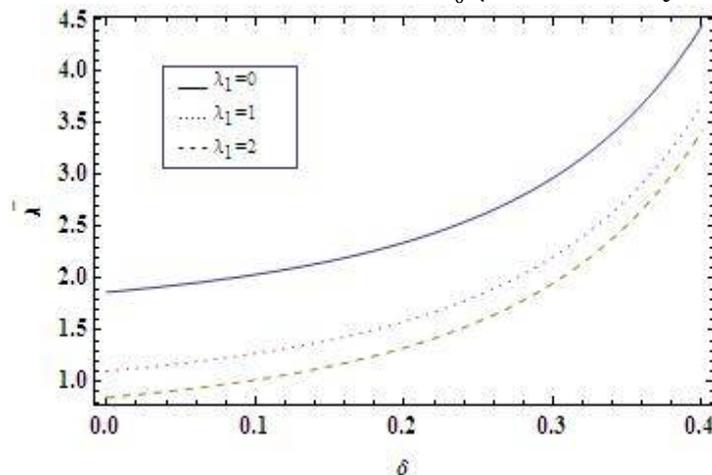


Fig 4: Variation of flow resistance $\bar{\lambda}$ with δ for different λ_1 ($d = 0.2, L = 1, L_0 = 0.4, Q = 0.1$)

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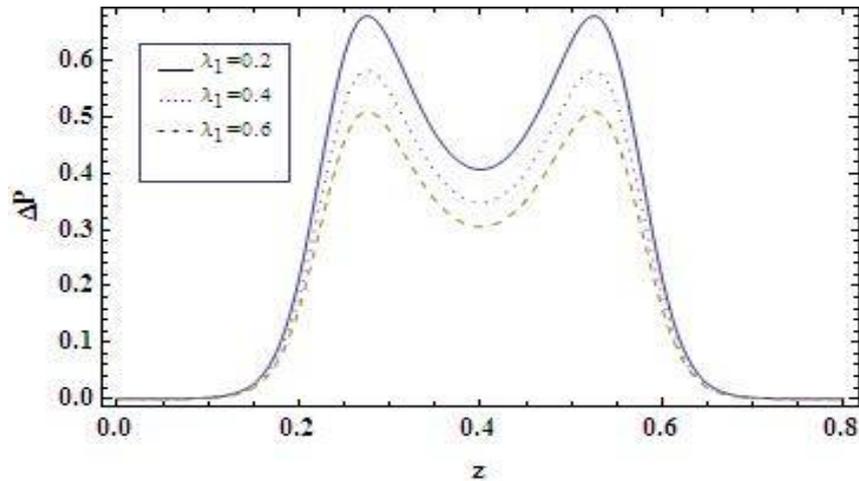


Fig 5: Pressure drop variation for different values of λ_1 ($d = 0.2, L = 1, L_0 = 0.4, Q = 0.1, \delta = 0.2$)

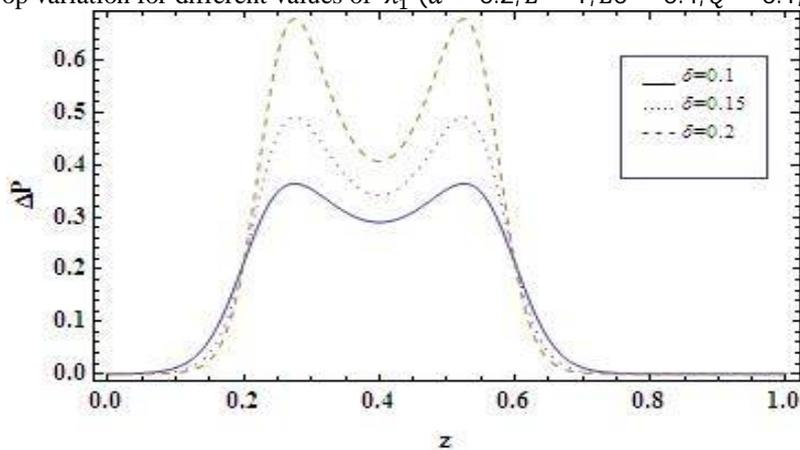


Fig 6: Pressure drop variation for different values of δ ($d = 0.2, L = 1, L_0 = 0.4, Q = 0.1, \lambda_1 = 0.2$)

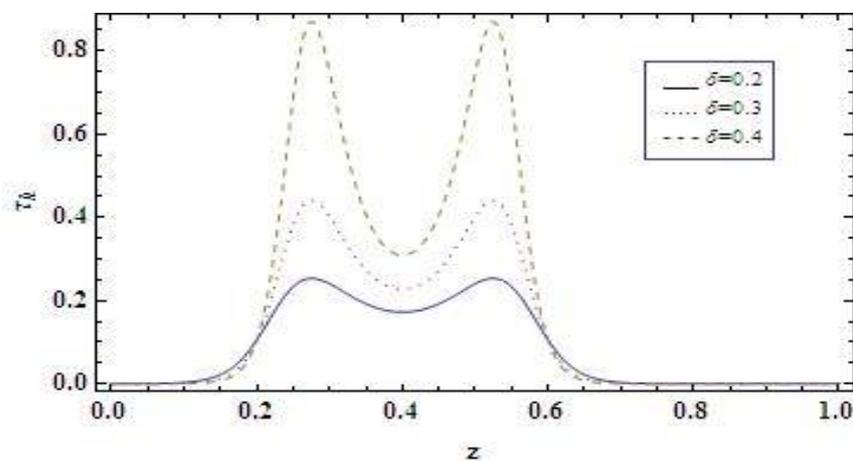


Fig 7: Variation of wall shear stress τ_h with δ ($d = 0.2, L = 1, L_0 = 0.4, Q = 0.1, \lambda_1 = 0.2$)

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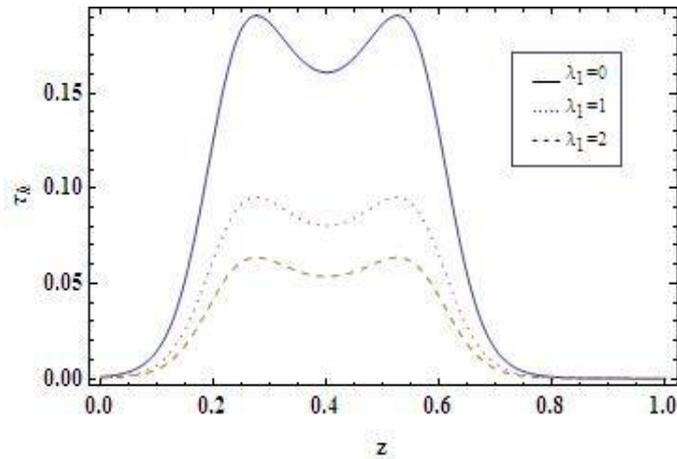


Fig 8: Variation of wall shear stress τ_h with λ_1 ($d = 0.2, L = 1, L_0 = 0.4, Q = 0.1, \delta = 0.2$)

IV. CONCLUSION

A mathematical model of Jeffrey fluid flow through an overlapping stenosis is presented. Closed form solutions are obtained for resistance to the flow and wall shear stress. It is analyzed that the resistance to the flow increases with the height and length of the stenosis but decreases with Jeffrey fluid parameter. The pressure drop and wall shear stress increases with stenosis height but decreases with Jeffrey fluid parameter.

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